Intro for Causality

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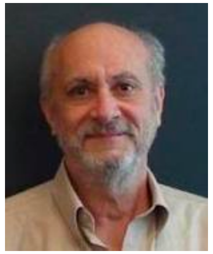
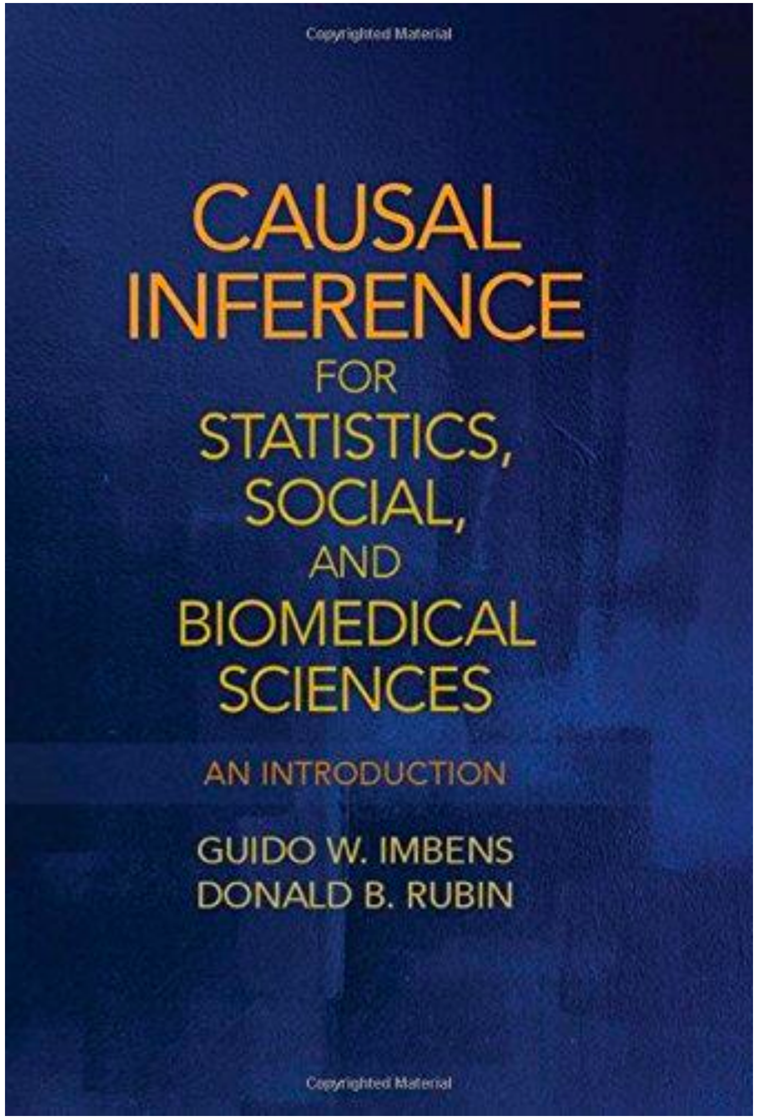
Two branches

- Donald Rubin
- potential outcome
- goal: causal effect

$$\delta_u = Y_{t_u} - Y_{c_u}$$

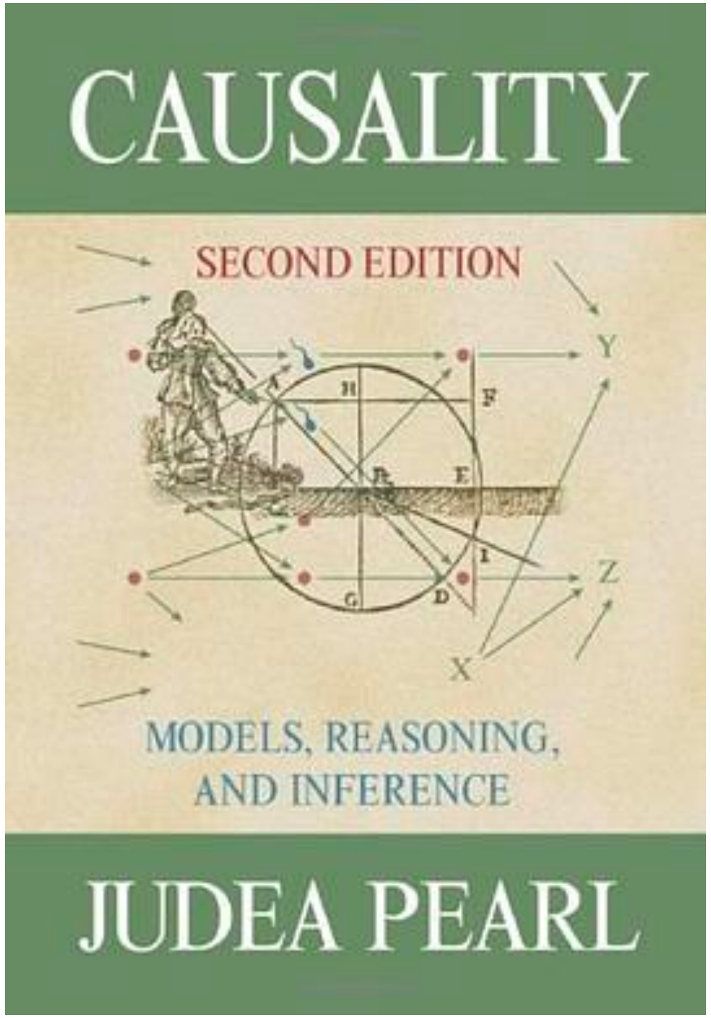
- bio-stats & bio-medical



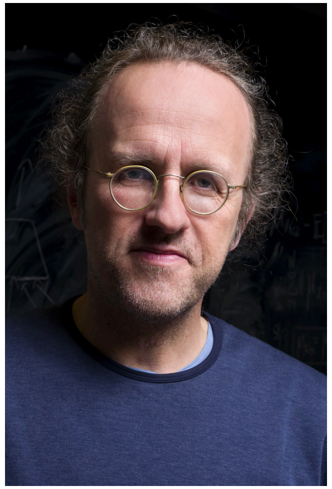


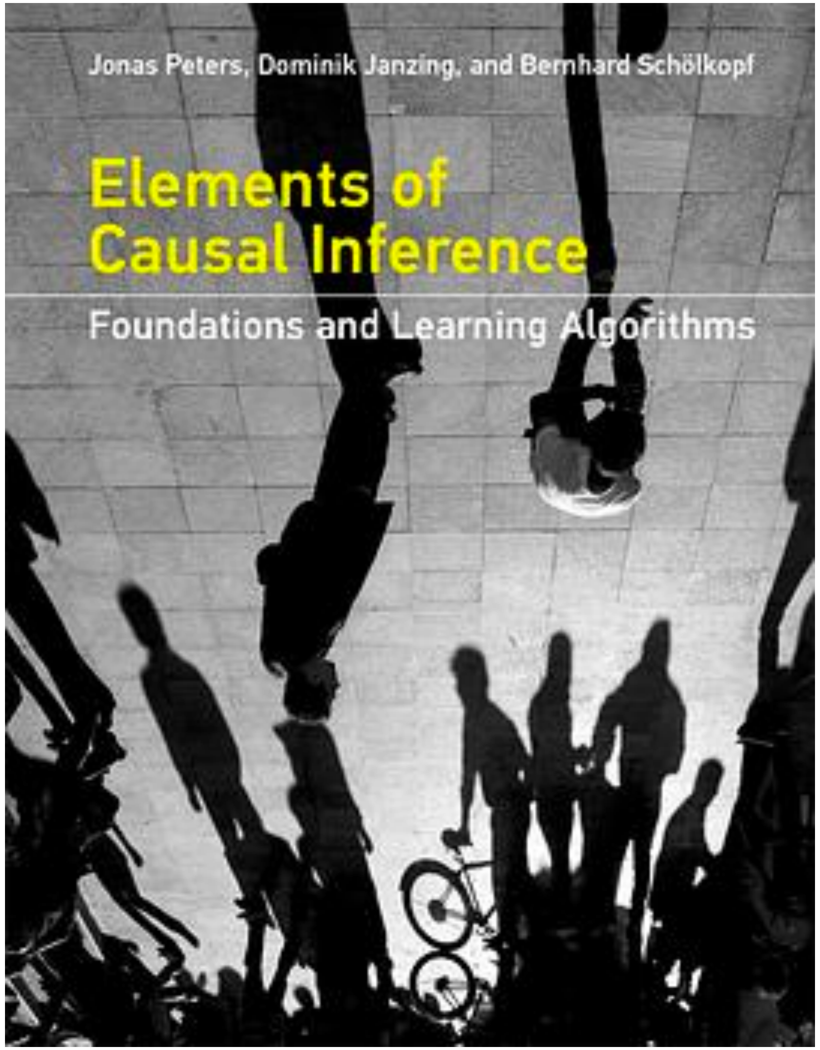
Two branches

- Judea Pearl
- Directed Acyclic Graph (DAG)
- causal discovery
- stats & ml

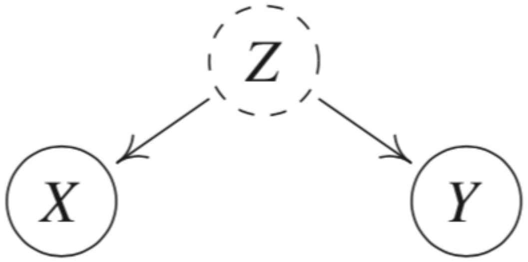


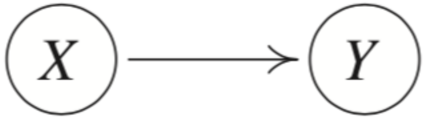
- Bernhard Schölkopf

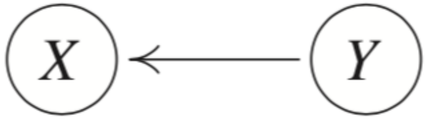




Confounder







$$X \perp\!\!\!\perp Y \mid Z$$

$$X \not\perp\!\!\!\perp Y$$

Structral Causal Model (SCM)

- A → T

$$A := N_A,$$
$$T := f_T(A, N_T)$$

- independence of cause and mechanism:
  - N_A $\perp\!\!\!\perp$ N_T
  - p(a,t) = p(a)p(t|a)

connection with SSL

- Semi-supervised learning used unlabeled X to help
- if $P_{\mathbf{X}}$ and $P_{Y|\mathbf{X}}$ are indeed independent,
- then SSL won't help

- therefore, all cases where SSL helps is *anti-causal*

$p(a,t) = p(a)p(t|a)$ or $p(t)p(a|t)$?

- IF
- **intervening** on A has changed T , but intervening on T has not changed A
- THEN
- we think A → T

Intervention

- C → E (cause → effect)

$$C := N_C$$
$$E := 4 \cdot C + N_E,$$

with $N_C, N_E \overset{\text{iid}}{\sim} \mathcal{N}(0,1)$, and graph $C \to E$. Then,

$$P_E^{\mathfrak{C}} = \mathcal{N}(0,17) \neq \mathcal{N}(8,1) = P_E^{\mathfrak{C};do(C:=2)}$$

$$P_C^{\mathfrak{C};do(E:=2)} = \mathcal{N}(0,1) = P_C^{\mathfrak{C}}$$

Counterfactual

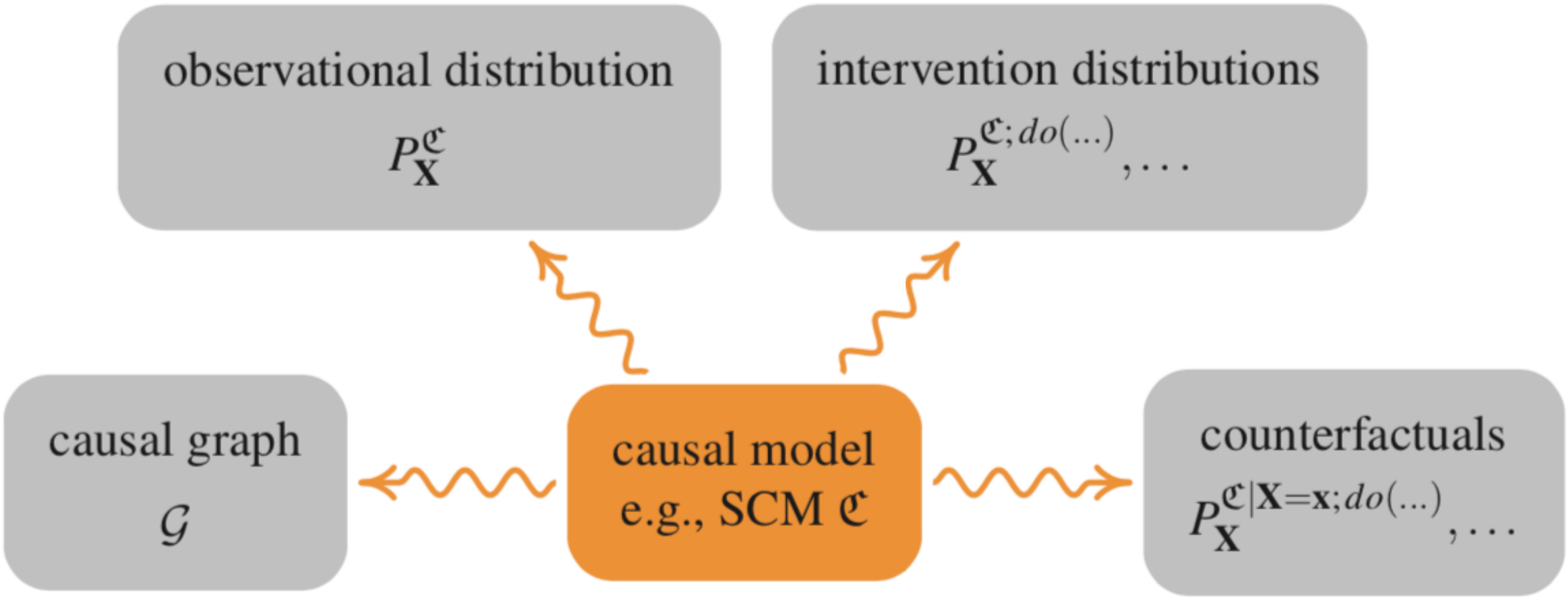
$$T \rightarrow B$$

$$\mathfrak{C}: \begin{array}{lcl} T & := & N_T \\ B & := & T \cdot N_B + (1-T) \cdot (1-N_B) \end{array}$$

$$N_B \sim \mathrm{Ber}(0.01)$$

- T = 1: with treatment
- N_B = 0: normal patient N_B = 1: rare patient
- B = 0: healthy B = 1: blind

$$P^{\mathfrak{C}|B=1,T=1;do(T:=0)}(B=0) = 1\,.$$


observational distribution
$P_{\mathbf{X}}^{\mathfrak{C}}$
intervention distributions
$P_{\mathbf{X}}^{\mathfrak{C};do(...)}, \ldots$
causal graph
$\mathcal{G}$
causal model
e.g., SCM $\mathfrak{C}$
counterfactuals
$P_{\mathbf{X}}^{\mathfrak{C}|\mathbf{X}=\mathbf{x};do(...)}, \ldots$

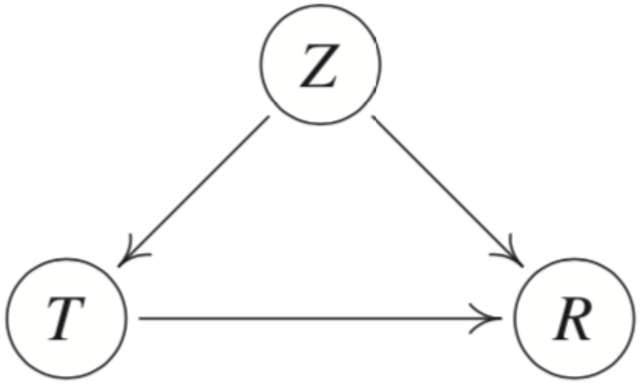
Simpson's paradox

	Overall	Patients with small stones	Patients with large stones
Treatment a: Open surgery	78% (273/350)	**93%** (81/87)	**73%** (192/263)
Treatment b: Percutaneous nephrolithotomy	**83%** (289/350)	87% (234/270)	69% (55/80)

conditional prob compare:

$$P^{\mathfrak{C}}(R = 1 \mid T = A) - P^{\mathfrak{C}}(R = 1 \mid T = B) = 0.78 - 0.83,$$

- Z: size of the stone
- R: whether recovery



- instead of compare conditional probability
- we should compare intervention probability:

$$\mathbb{E}^{\mathfrak{C}_A} R = P^{\mathfrak{C}_A}(R=1) = P^{\mathfrak{C};do(T:=A)}(R=1) \qquad \mathbb{E}^{\mathfrak{C}_B} R = P^{\mathfrak{C}_B}(R=1) = P^{\mathfrak{C};do(T:=B)}(R=1).$$

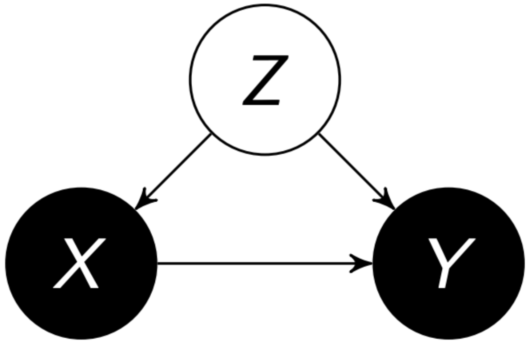
$$p^{\mathfrak{C};do(T:=t)}(r) = \sum_z p^{\mathfrak{C}}(r|z,t)p^{\mathfrak{C}}(z) \neq \sum_z p^{\mathfrak{C}}(r|z,t)p^{\mathfrak{C}}(z|t) = p^{\mathfrak{C}}(r|t).$$

$$P^{\mathfrak{C}_A}(R=1) \approx 0.93 \cdot \frac{357}{700} + 0.73 \cdot \frac{343}{700} = 0.832.$$

$$P^{\mathfrak{C}_A}(R=1) - P^{\mathfrak{C}_B}(R=1) \approx 0.832 - 0.782$$

$$P^{\mathfrak{C}}(R=1\,|\,T=A) - P^{\mathfrak{C}}(R=1\,|\,T=B) = 0.78 - 0.83,$$

Most important case: confounder correction



$$p(y|do(x)) = \sum_z p(y|x,z)p(z) \neq \sum_z p(y|x,z)p(z|x) = p(y|x)$$

If equal, then X -> Y

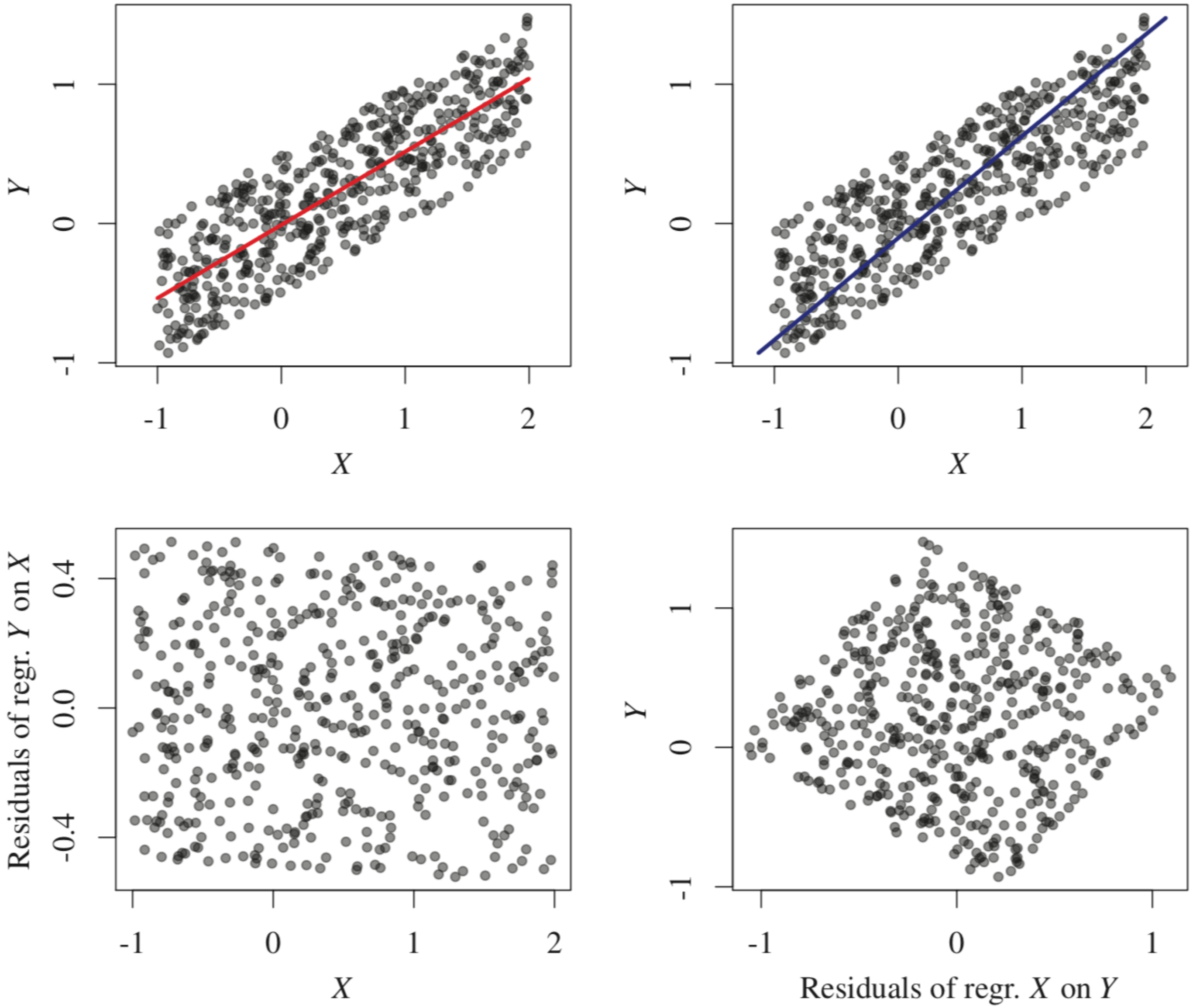
Learning Cause-Effect

- Identifiability
- additional assumptions are required

Structure Identification

- Given (X, Y) ~ dataset

1. Regress Y on X; that is, use some regression technique to write Y as a function $\hat{f}_Y$ of X plus some noise.
2. Test whether $Y - \hat{f}_Y(X)$ is independent of X.
3. Repeat the procedure with exchanging the roles of X and Y.
4. If the independence is accepted for one direction and rejected for the other, infer the former one as the causal direction.



Y
X
Residuals of regr. Y on X
Residuals of regr. X on Y

Alternative approach

- compare independence:
- p(x) $\perp\!\!\!\perp$ p(y|x) or p(y) $\perp\!\!\!\perp$ p(x|y) ?

Supervised learning approach

$$(\mathcal{D}_1, A_1), \ldots, (\mathcal{D}_n, A_n).$$

$$\mathcal{D}_i = \{(X_1, Y_1), \ldots, (X_{n_i}, Y_{n_i})\} \qquad \underline{A_i \in \{\rightarrow, \leftarrow\}}$$

Thank you for listening